Physics 302/572: Electromagnetic Theory II

Read: Griffiths 10.2-10.3

“G, PS” refer to Griffiths and Pollack & Stump books respectively. Problems with stars are not for credit and will NOT be graded.

Homework 7

Exercise 1 (G 10.11)

Suppose $\vec{J}(\vec{r})$ is constant in time. In this case it is easy to show that the charge density is a linear function of time $\rho(\vec{r}, t) = \rho(\vec{r}, 0) + \dot{\rho}(\vec{r}, 0)t$. Show that

$$\vec{E}(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}', t)}{R^2} \hat{R} \, d\tau';$$

that is, Coulomb’s law holds, with the charge density evaluated at the non-retarded time.

Exercise 2 (G 10.18)

Suppose a point charge $q$ is constrained to move along the $x$ axis. Show that the fields at points on the axis to the right of the charge are given by

$$\vec{E} = \frac{q}{4\pi \epsilon_0} \frac{1}{R^2} \left( \frac{c + v}{c - v} \right) \hat{x}, \quad \vec{B} = 0.$$

What are the fields on the axis to the left of the charge?

*Exercise 3 (G 10.19)

Using the formulas for electric and magnetic fields of a point charge moving with a constant velocity

a) Calculate the electric field at a distance $d$ from an infinite straight wire carrying a uniform line charge $\lambda$, moving at a constant speed $v$ down the wire.

b) Calculate the magnetic field of this wire.
Exercise 4 (G 10.24)

One particle, of charge $q_1$, is held at rest at the origin. Another particle, of charge $q_2$, approaches along the $x$ axis, in hyperbolic motion:

$$x(t) = \sqrt{b^2 + (ct)^2};$$

it reaches the closest point, $b$, at time $t = 0$, and then returns out to infinity.

a) What is the force $F_2$ on $q_2$ (due to $q_1$) at time $t$?

b) What total impulse ($I_2 = \int_{-\infty}^{\infty} F_2 \, dt$) is delivered by $q_2$ to $q_1$?

*c) What is the force $F_1$ on $q_1$ (due to $q_2$) at time $t$?

*d) What total impulse ($I_1 = \int_{-\infty}^{\infty} F_1 \, dt$) is delivered by $q_1$ to $q_2$?

Answer: $I_2 = -I_1 = \frac{q_1 q_2}{4 \varepsilon_0 b c}$.

Exercise 5 (PS 15.21)

Calculate the Poynting vector and energy density of the electromagnetic field of a charged particle moving with constant velocity. Show that the field energy is carried along with the particle.