Physics 503: Methods of Mathematical Physics

Read: CKP chapter 2, sections 2-1 — 2-5.

“CKP” refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 2

Exercise 1 (CKP, page 29, problem 2)
Verify the Cauchy-Riemann equations for \((1 - z^2)^{1/2}\). At what points this function has singularities?

*Exercise 2 (CKP, page 29, problem 2)
Prove in an easy way that \((x^2 + y^2)^{1/4} \cos \left(\frac{1}{2} \arctan \frac{y}{x}\right)\) is harmonic.

Exercise 3 (CKP, page 30, problem 7)
If \(u\) and \(v\) are expressed in terms of polar coordinates \((r, \theta)\), show that the Cauchy-Riemann equations can be written

\[
\frac{u_r}{r} = \frac{1}{r} v_\theta, \quad \frac{1}{r} u_\theta = -v_r.
\]

Exercise 4 (CKP, page 36, problem 3)
Show in an easy way that the integral of each of the following expressions around the circle \(|z| = 1/2\) vanishes:

\[
a) \quad \frac{z + 1}{z^2 + z + 1}, \quad b) \quad e^{z^2} \ln(1 + z), \quad c) \quad \arcsin z.
\]

Exercise 5 (CKP, page 40, problem 1)
Use Cauchy’s integral formula to evaluate the integral around the unit circle \(|z| = 1\) of

\[
a) \quad \frac{\sin z}{2z + 4}, \quad b) \quad \frac{\ln(z + 2)}{z + 2}, \quad c) \quad \frac{z^3 + \text{arcsinh}(z/2)}{z^2 + 12 + 4i}.
\]
*Exercise 6

Find the principal value of the integral $\int_C \frac{\sin z}{z^2} \, dz$ where counterclockwise contour $C$ is a square $ABDF$ with $A = 0$, $B = 2\pi$, $D = 2\pi(1 + i)$, and $F = 2\pi i$.

Exercise 7 (CKP, page 43, problem 1)

Find the maximum for $|z| \leq 1$ of functions

$$a) \ |z^2 + 2z + i|, \quad b) \ |\sin(z)|, \quad c) \ |\arcsin \frac{z}{2}|.$$