Homework 12

Reading
JJS 3.1-3.3, 3.5-3.7.

Problem 1
What is the expectation value of \( \frac{1}{2}(L_x L_y + L_y L_x) \) in the state with angular part of the wave function given by \( Y^2_3(\theta, \phi) \)?

Problem 2
A \( D_2 \) is known to be in the state
\[
\psi(\theta, \phi) = \frac{3Y^1_1 + 4Y^3_7 + Y^1_7}{\sqrt{26}}.
\]
What values of \( L \) and \( L_z \) will measurement find and with what probabilities these values occur?

Problem 3
At a given instant of time, a rigid rotator is in the state
\[
\psi(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \sin \phi \sin \theta.
\]
a) What possible values of \( L_z \) will measurement find and with what probability will these values occur?
   b) What is \( \langle L_z \rangle \) for this state?
   c) What is \( \langle L^2 \rangle \) for this state?

Problem 4
How many linearly independent singlets can be constructed from four spin 1/2? From 6 spin 1/2?

Problem 5
What are the possible values of \( l \) for
   a) Four \( p \) electrons?
   b) Three \( p \) and one \( f \) (\( l = 3 \)) electrons?

Problem 6
The energy of a rigid molecule, free to rotate about its center of mass, is given by
\[
E = \frac{L^2_x + L^2_y}{2I_1} + \frac{L^2_z}{2I_3},
\]
where moments of inertia \( (I_1, I_2, I_3) \) are evaluated in principal axis with \( I_1 = I_2 \) and the origin at the center of mass.
Find the eigenenergies and eigenfunctions for this molecule.
Problem 7
Find the energy levels of a particle in a spherical box of radius $R$ in the $l = 0$ sector.